SYLLABUS

FOR

PH.D. COURSE WORK

IN MATHEMATICS

With effect from the academic session 2020-2021

KAZI NAZRUL UNIVERSITY

ASANSOL-713 340

WEST BENGAL

Course Structure for Ph.D. Program in Mathematics

Duration of Ph.D. course work in Mathematics shall be one year with two Semesters. Total credits for this course would be 12. There will be 8 credits in Semester I comprising of two units, one is Compulsory unit and another is Elective unit. The elective unit shall be offered by the prospective Supervisor concerned. In Semester II, there will be two compulsory units each of 2 credits.

The distribution of credits of the Course Work syllabus shall be as follows:

Detail syllabus of the Ph.D. pre-registration course work in mathematics

PHDMATHC101: RESEARCH METHODOLOGY

Total Marks: 50 (10 marks reserved for internal assessment) **Credit: 4**

Research: Definition, importance, meaning and characteristics. Steps in Research. Researchproblem: identification, selection and formulation.
Data: Definition, types, sources, data collection methods. Review of literatures and Bibliography.Research report: Types, contents, styles and steps in drafting. Editing the final draft, way of writing research papers, subject classifications and write-up of Thesis. Significance of Impact factor, citation index, science citation index, IST, SCOPUS etc.
Review of articles and Research proposal. An overview of Mathematical Reviews (Author's index, subject index) with subject classifications.

Basics of Computer Operating System: Using Windows – Directory structures – command structure (Document preparation, EXCEL, Power Point Presentation).

Use of Latex software, Preparation of Manuscript using Latex(Typing of Research Paper and Seminar presentation).

COURSE II: ELECTIVE UNIT

(One Elective Unit of 4 credits to be offered by the prospective Supervisor concerned)

For each Elective Unit Total Marks: 50 (10 marks reserved for Internal Assessment)& Credit: 4

PHDMATHC102: Computational Techniques using Mathematica and Matlab

Basics of Mathematica software. 2 D and 3 D Graphs . Basics of Calculus. Ordinary Differential Equations. Partial Differential Equations and Boundary Value Problems.

Programming technique using Mathematica. Linear and Nonlinear Integral Equations. Matrix operations in MATLAB. Solution of Equations. Curve-fitting. Numeral Integration.

MATLAB Programming.

PHDMATHC103:Inventory Control

Inventory control of style goods and perishable items. Production planning for unreliable production systems. Integrated production, quality and maintenance models. Production planning and inventory control in fuzzy environment. Supply chain ¡V definition, decision phases, process view. Centralized supply network versus decentralized operation. Coordination. Bullwhip effect. Multi-echelon supply chains. Simple models of supply chain management. Solving inventory/supply chain management problem using Genetic Algorithms (GAs).

PHDMATHC104:Commutative Algebra

Regular Sequences and Depth: Regular Sequences, Grade and Depth, Depth and Projective Dimension, Some Linear Algebra, Graded rings and modules. The Koszul Complex. The Eagon-Northcott complex.

Cohen-Macaulay Rings: Cohen ¡V Macaulay rings and modules, Regular rings and normal rings, Complete Intersections.

Determinantal Rings: Graded Hodge Algebras, Starightening Laws on Posets of Monors, Properties of Determinantal Rings.

PHDMATHC105:Integral Equations

Basic definitions, regular, singular, hypersingular integral equations. Occurrence of integral equations in classical mechanics, ordinary differential equations, partial differential equations. Occurrence in continuum mechanics (elasticy, fluid mechanics). Singular integral equations, Abel integral equations, solutions, Cauchy singular integral equations, solutions, applications. Hypersingular integral equations, solution of simple hypersingular integral equations,

applications. Dual integral equations. Solution for trigonometric function kernels, applications.

PHDMATHC106:Modules, Rings, Groups and Categories

Tensor Product of Modules, Categories, Functions and Natural Transformations, Exact sequences, Projective, Injective and Flat Modules, Localization, Group Representation Theory.

PHDMATHC107:Theory of Semi groups & Lattice Theory

Introduction : Basic Definitions and Results : Congruences, Rees congruences, Ideals, Homomorphisms etc. Green¡¦s Equivalence Relations and Regular Semigroups. Completely Regular Semigroups : Characterization of completely regular semigroups as union of groups, semilattices of groups, Clifford Semigroups, Intra-regular Semigroups, Orthodox Semigroups, Inverse Semigroups etc.

Types of Lattices, Postulates for Lattices, Structure and Representation Theory Complete lattices, Lattice ordered groups, lattice ordered monoids, lattice ordered rings, vector lattices.

PHDMATHC108: Integral Transform:

Fourier Transforms: Fourier integral theorem, Definition and properties of Fourier Transforms, Fourier Transforms of Derivatives, Fourier Transforms of some useful functions, Fourier sine and cosine transforms, Inversion formula of Fourier Transforms, Convolution.

Theorem, Parseval's relation, Application of Fourier transforms to Heat, Wave and Laplace equations.

Laplace Transforms: Definition and properties of Laplace transforms, sufficient conditions for the existence of Laplace Transform, Laplace Transform of some elementary functions, Laplace Transforms of the derivatives, , Initial and final value theorems, Convolution theorems, Inverse of Laplace Transform, Bromwich integral theorem, Application to Ordinary and Partial differential equations.

PHDMATHC109:Fuzzy Sets and Fuzzy Logic:

Fuzzy sets-basic definitions, alpha-level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, Cartesian products, algebraic products, bounded sum and difference, t-norms and t-conorms.

The extension principle- the Zadeh's extension principle, image and inverse image of fuzzy sets, fuzzy numbers, elements of fuzzy arithmetic.

Fuzzy relations and fuzzy graphs, composition of fuzzyrelations, min-max composition and its properties, fuzzy equivalence relations, fuzzy graphs.

Fuzzy logic, fuzzy propositions, fuzzy quantifiers, linguistic variables, inference from conditional fuzzy propositions, compositional rule of inference.

PHDMATHC110: Nonlinear Wave Theory

Linear waves: Linear wave equation, dispersion relation, dispersive and dissipative waves, group velocity

Nonlinear equations of evolution: Effect of nonlinearity, diffusive waves, dispersive waves, solitary wave, soliton.

Soliton interaction: Schrodinger equation, KdV equation and their interrelationship, time independence of the eigenvalues of the Schrodinger equation and determination of scattering parameters, inverse scattering problem, soliton solution of KdV equation, soliton interaction, continuous eigenvalues of the Schrodinger operator

Solitary wave theory: Dispersion and dissipation, types of travelling wave solutions, nonanalytic solitary wave solution, analysis of Adomian decomposition method and variational iteration method on nonlinear partial differential equations

PHDMATHC111: Fractional Calculus & Applications

The Riemann Liouville Fractional Calculus: Fractional Integrals of some functions namely binomial function, exponential, the hyperbolic and trigonometric functions, Bessel's functions, Hyper-geometric function and the Fox's H-function. Dirichlet's Formula, Derivatives of the Fractional Integral and the Fractional Integral of Derivatives. Laplace Transform of the Fractional integral, Leibniz's Formula for Fractional Integrals. Derivatives, Leibniz's Formula of Fractional Derivatives.

The Weyl Fractional Calculus – Definition of Weyl Fractional Integral Weyl Fractional Derivatives, A Leibniz Formula for Weyl Fractional Integral and simple applications.

Fractional Differential Equations: Introduction, Laplace Transform, Linearly Independent Solutions, Solutions of the Homogeneous Equations, Solution of the Non-homogeneous Fractional Differential Equations, Reduction of Fractional Differential Equations to ordinary differential equations. Semi Differential equations.

PHDMATHC112: Dynamical Systems

History of dynamical system, mathematical definition, different types of dynamical systems with examples, phase variable and phase space, continuous and discrete dynamical systems, Flows and maps, orbits, fixed points, periodic points and their stabilities, Attractors and Repellors.

Phase plane analysis, hyperbolic concept of hyperbolicity, stable, unstable and center subspaces. Lyapunov and asymptotic stability,Local and global stability, Hartmann-Grobman theorem (statement only), stable manifold theorem, Lyapunov function, Lyapunov theorem on stability, periodic orbits, limit cycles, attracting and invatriant sets, Poincare-Bendixson theorem, Poincare map, Lienard's theorem (statement only) and applications. Bifurcation theory, Saddle-Node, Pitch-Fork and Transcritical bifurcations for one-dimensional continuous systems, Hopf-bifurcation, Analysis of Lorentz system.

Some important maps: Logistic map, Tent map, Baker map, Shift map and their properties.

PHDMATHC113: Prey-Predator Model

Single species population model, Multi species model, Prey Predator model in terms of differential equations. Stability of Prey Predator Model

PHDMATHC114:Optimization Techniques

Integer Programming: Standard form of Integer Programming, The concept of cutting plane for linear integer programs, Gomory's cutting plane method, Gomory's All-Integer Programming Method, Branch-and-Bound Algorithm for general integer programs.

Optimal Control: Performance indices, Methods of calculus of variations, simple optimal problems of mechanics.

Non-linear Programming: Formulation of Non-linear programming problem, Unconstrained optimization, Optimization with equality constraints, Kuhn-Tucker conditions for constrained optimization.

Quadratic Programming: Wolfe's modified simplex method, Beale's method. Convex Programming.

PHDMATHC115:Advanced Complex Analysis

The Functions M(r), A(r), Hadamard Theorem on Growth of log M(r), Schwarz Inequality, Borel-Caratheodory Inequality.

Entire functions, Growth of an entire function, Order and type and their representations in terms of the Taylor Coefficients, Distribution of zeros, Schottky's theorem (without proof), Picard's Little theorem, Weierstrass Factor theorem, The exponent of convergence of zeros, Hadamard factorization theorem, Canonical product, Borel's first theorem, Borel's second theorem (statement only).

Analytic continuation, Natural boundary, Analytic element, Global analytic function, Concept of analytic manifolds, Multiple valued conditions, Branch points and Branch cut, Riemann surfaces.

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PHDMATHC116:Advanced Functional Analysis

Topological vector spaces, Local base and its properties, Separation properties, Locally compact topological vector spaces and its dimension. Convex Hull and representation Theorem, Extreme points, Symmetric sets, Balanced sets, absorbing sets, Bounded sets in topological vector space. Linear operators over topological vector space, Boundedness and continuity of Linear operators, Minkowski functional, Hyperplanes, Separation of convex sets by Hyperplanes, Krein-Milman Theorem on extreme points.

Locally convex topological vector spaces, Criterion for normability, Semi norms, Generating family of semi norms in locally convex topological vector spaces. Barreled spaces and Bornological spaces. Criterion for locally convex topological vector spaces to be (i) Barreled and (ii) Bornological.

Strict convexity and uniformly convexity of a Banach space. Uniform Convexity of a Hilbert Space. Reflexivity of a uniformly convex Banach space, Weierstrass approximation theorem in C [a,b].

PHDMATHC117:Advanced Topology

Countability and Separation Axioms: Countability axioms, The separation axioms, Equation spaces, Lindelöf spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze extension theorem.

Nets and Filters: Directed sets, Nets and Subnets, Convergence of a Net, Ultranets, Partially ordered sets and filters, Convergence of a filter, Ultrafilters, Basis and subbase of a filter, Nets and Filters in Topology.

Tychonoff Theorem & Compactification: Tychonoff theorem, Completely regular spaces, Local compactness, One-point compactification, Stone-Cech compactification.

Metrization: Urysohn Metrization theorem, Topological imbedding, Imbedding theorem of a regular space with countable base, Partitions of unity, Topological m-Manifolds, Imbedding theorem of a compact m-Manifold in R^n. Local finiteness, Nagata-Smirnov Metrization theorem, Paracompactness, Stone's theorem, Local metrizability, Smirnov Metrization theorem, Uniform spaces.

Compactness in metric spaces, Equicontinuity, Pointwise and compact convergence, The compact-open topology, Stone-Weierstrass theorem, Ascoli's theorem, Baire spaces, A nowhere differentiable function.

PHDMATHC118:Differential Geometry and Manifolds

Definition and examples of differentiable manifolds. Tangent spaces. Jacobian map. One parameter group of transformation. Lie derivatives. Immersions and imbedding. Distributions.

Exterior algebra. Exterior derivative.

Topological groups. Lie groups and Lie algebras. Product of Two Liegroups. One parameter subgroup and exponential maps. Examples of Liegropus. Homomorphism and Isomorphism.

Lie transformation groups, General linear group.

Principal fibre bundle. Linear frame bundle. Associated fibre bundle. Vector bundle. Tangent bundle. Induced bundle. Bundle homeomorphisms.

PHDMATHC119:Field Theory

Field Extensions: Algebraic and Transcendental Extensions, Finite Extensions, Algebraic Closure of a field, Algebraically Closed Field, Splitting Field of a polynomial, Normal Extensions, Separable Extensions, Impossibility of some constructions by straightedge and compass.

Finite Fields and their properties, Galois group of automorphism and Galois Theory, Solution of polynomial equations by radicals, Insolvability of the general equation of degree 5 (or more) by radicals.

PHDMATHC120:Operator Theory

Bounded Linear Operators: Resolvent Set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Spectral properties of a bounded linear operator, Spectral mapping theorem for polynomials. Numerical range, Numerical radius, Convexity of numerical range, Closure of numerical range contains the spectrum, Relation between the numerical radius and norm of a bounded linear operator.

Banach Algebra: Definition of normed and Banach algebra and examples, Singular and non-singular elements, the spectrum of an element, The spectral radius.

Compact linear operators: Spectral properties of compact linear operators on a normed linear space, Operator equations involving compact linear operators, Fredholm alternative theorem, Fredholm alternative for integral equations. Spectral theorem for compact normal operators.

PHDMATHC201: REVIEW OF RESEARCH WORK

Total Marks: 50 (10 marks reserved for internal assessment) **Credit: 2**

The relevance of the research work and its perspective on the subjectconcerned – Possible waysof utilities for further research work.

Review of Literatures on the specific research area to be submitted by each scholar.

PHDMATHC202: RESEARCH AND PUBLICATION ETHICS

Total Marks: 50 (10 marks reserved for internal assessment) **Credit: 2**

Philosophy and Ethics: Introduction to philosophy: Definition, nature and scope, concept, branches. Ethics: definition, moral philosophy, nature of moral judgments and reactions.

Scientific conduct: Ethics with respect to science and research, Intellectual honesty and research integrity. Scientific misconduct: Falsification, fabrication, and Plagiarism (FFP). Redundant publications: duplicate and overlapping publications, salami slicing. Selective reporting and misrepresentation of data.

Publication Ethics: definition, introduction and its importance. Best practices/standards setting initiatives and guidelines: COPE, WAME, etc. Conflicts of interest. Publication misconduct: definition, concept, problems that lead to unethical behavior and vice versa, types. Violation of publication ethics, authorship and contributor ship. Identification of publication misconduct, complaints and appeals. Predatory publishers and journals.

Open access publishing: Open access publications and initiatives. SHERPA/RoMEO online resource to check publisher copyright and self archiving policies. Software tool to identify predatory publications developed by SPPU. Journal finder/Journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

Use of plagiarism detection software like Turnitin, ithenticate, Urkund and other open source software tools.

Databases: Indexing databases, citation databases, Web of Science, Scopus, etc. Research Metrics: Impact factor of Journals as per Journal Citation Report, SNIP. SJR, IPP, Cite Score. Metrics: h-index, g-index, i-10index, altmetrics.